Int. J. Multiphase Flow Vol. 8, No. 2, pp. 125-146, 1982
Printed in Great Britain.

0301-9322/82/020125-22$03.00/0
Pergamon/Elsevier

ON THE MOTION OF PARTICLES IN TURBULENT DUCT FLOWS

S. L. LEE
Department of Mechanical Engineering, State University of New York at Stony Brook, Stony Brook, NY 11794, U.S.A.

and

F. DURST
Sonderforschungsbereich 80, Universität Karlsruhe, 7500 Karlsruhe 2, Federal Republic of Germany

(*Received* 29 *January* 1980; *in revised form* 1 *July* 1981)

Abstract—Existing knowledge on particle deposition rates on walls from turbulent pipe and channel flows is summarized and it is shown that discrepancies exist between experimental and theoretical findings. To contribute to the existing experimental information, laser Doppler measurements are reported of the flow field of a glass particle-air two-phase flow. The results reveal certain seemingly peculiar behaviors of the particles which obviously defy the predictions of the conventional analyses of turbulent two-phase suspension flows.

In an accompanying approximate, yet pragmatic theoretical approach, an attempt is made to find a rational basis for the explanation of these experimentally observed particle behaviors. It is shown for the particles in the present study, there exists a limiting size above which their response to the agitation of the fluctuating motion of the surrounding fluid could be treated as if the flow were laminar. On this rational basis, these experimentally observed particle behaviors can then be qualitatively explained by the existing theory of particle excursion in a laminar shear flow field.

Reported also is a suggestion to extend the present analysis to a dispersion of particles of multiple sizes.

1. INTRODUCTION

The deposition of solid particles or liquid droplets from a turbulent particle dispersion flow to channel walls is a problem of fundamental importance in a variety of technical areas. Although a large number of papers has become available on this subject, reliable results from carefully planned experiments are scarce and only provide information on the wall deposition without contributing to an answer fo the question of the mechanisms in the flow which are responsible for the particle motion towards the wall. This motion is assumed in most theoretical treatments to be due to turbulent diffusion. However, recent measurements of particle deposition rates at the wall of a vertical duct flow, see figure 1, are in disagreement with results of theoretical treatments computed on the basis of a gradient-driven particle transport. The present paper draws attention to other experimental results for particle deposition rates from air flows obtained by Alexander & Coldren (1951) and Cousins & Hewitt (1968) for large droplets and by Friedlander & Johnstone (1951) for small solid particles. It is pointed out by the authors that these data also cannot be compared with existing theories on the particle transport in turbulent flows. New approaches are needed on the basis of a more comprehensive understanding of the dynamics of the turbulent particle motion which leads to wall deposition. The present paper is an attempt in this direction. Existing experimental data are extended by LDA-studies of vertical, upward solid particle-gas two-phase flows and a practical approach is suggested to treat the motion of particles in turbulent flows in general and turbulent pipe flow in particular. The presented analysis for monodispersed particle size distributions is suggested to be extendable to polydispersed distributions.

2. LDA-MEASUREMENTS IN PARTICULATE TWO-PHASE FLOWS

2.1 *The LDA-equipment and the test-rig*

The basic ideas for LDA-velocity measurements in two-phase flows adopted in the present study have been developed by Durst & Zaré (1975) who showed that the light wave fronts, produced by two laser beams reflected on the smooth surface of a large body, interfere with

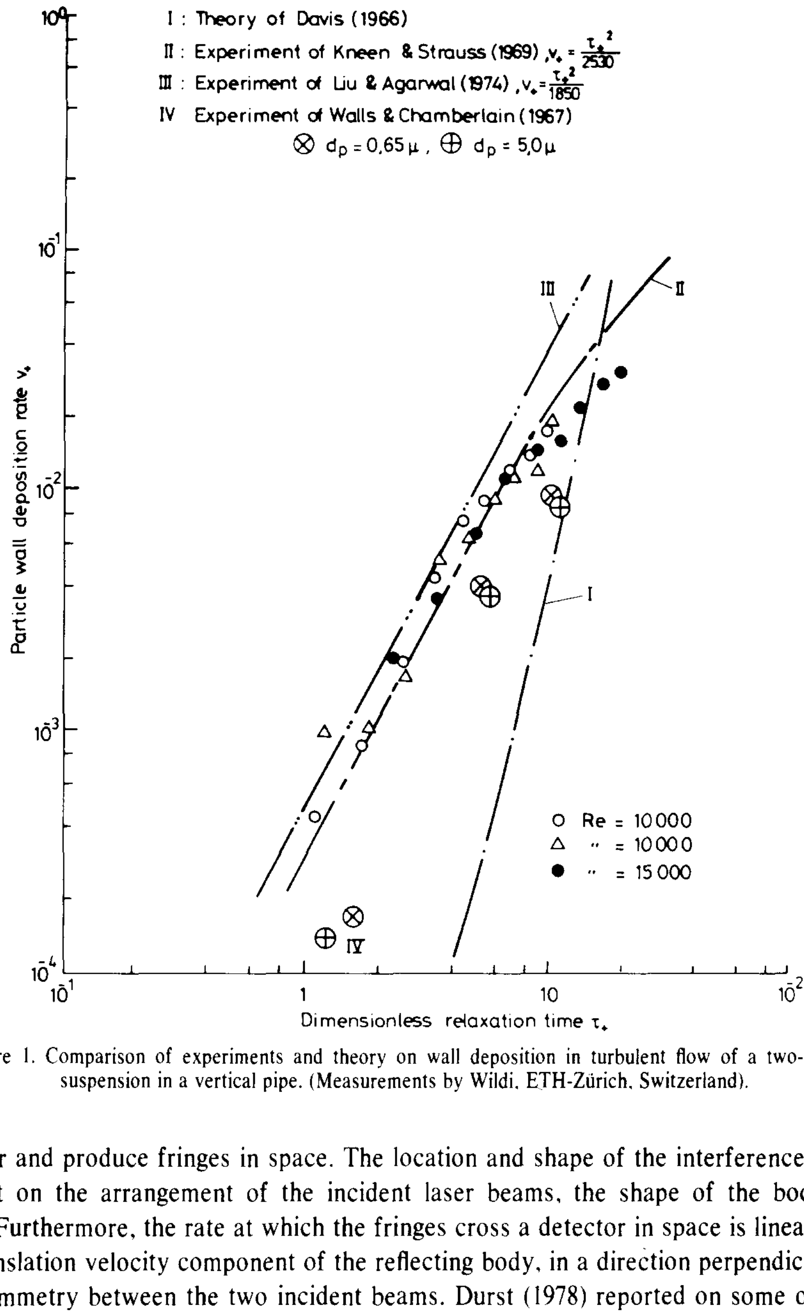


Figure 1. Comparison of experiments and theory on wall deposition in turbulent flow of a two-phase suspension in a vertical pipe. (Measurements by Wildi, ETH-Zürich, Switzerland).

each other and produce fringes in space. The location and shape of the interference pattern is dependent on the arrangement of the incident laser beams, the shape of the body and its location. Furthermore, the rate at which the fringes cross a detector in space is linearly related to the translation velocity component of the reflecting body, in a direction perpendicular to the line of symmetry between the two incident beams. Durst (1978) reported on some optical and electronic systems that had been designed to carry out local velocity measurements in particulate two-phase flows. One of these systems is shown in figure 2 which was employed in the present study. Figure 2(a) shows the optical system and figure 2(b) the processing electronics containing automatic filter banks which are used in conjunction with amplitude discriminators in order to separate those signals that are obtained from the fluid and the suspended larger particles.

The afore-mentioned filter banks have been designed for LDA-measurements in single-phase flows and extended to two-phase flow applications as described by Durst & Heidbreder (1979). Each of these filter banks consists of 15 band pass filters that permit automatic selection of the optimum filtering for LDA-signals within a range between 500 Hz and 50 MHz. An internal logic uses the output of the filters for the purpose of selecting the optimum filter. This yields filtered signals as indicated in figure 3.

In particulate two-phase flows, LDA-signals are obtained that differ in amplitude and

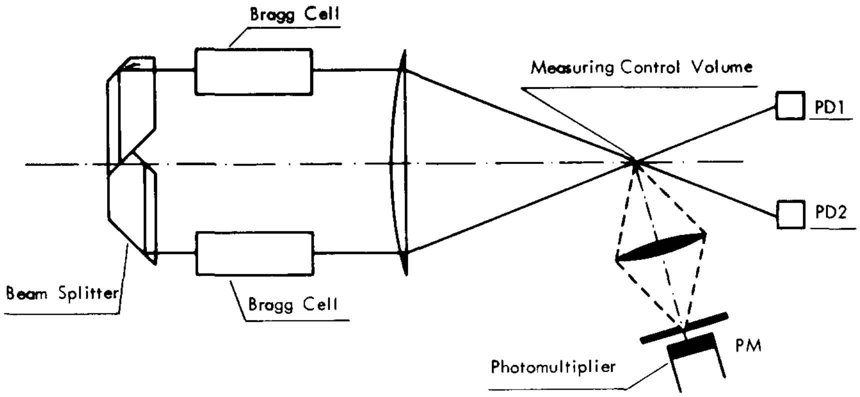


Figure 2(a). Optical arrangement I for two-phase flow LDA-measurements.

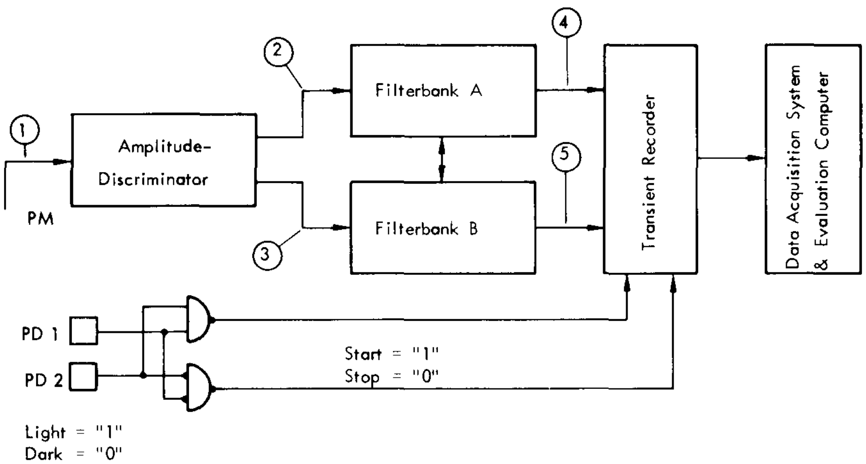


Figure 2(b). Block diagram of signal processing system for two-phase flow LDA-measurements.

modulation depth dependent on whether they are generated by small particles that closely follow the fluid motion or by large particulates. Such signals are indicated in figure 4 and the scheme of signal separation by amplitude discrimination and filtering are shown in figure 5. This sequence of signals demonstrates the functioning of the amplitude control in the electronic system given in figure 2(b) and also shows the functioning of the automatic filter banks *A* and *B* to produce the resulting signals *d* and *e*. These signals can be processed to obtain local information on the velocity fields of the two phases. In addition to the amplitude discrimination, the signals from the two photodiodes PD 1 and PD 2, as shown in figures 2(a) and 2(b), are used to separate the signals of large and small particles. The combined usage of the amplitude discriminator and the photodiodes causes the cross talk between signals from the small and large particles to be reduced to a great extent.

In order to test the signal processing electronics, simulated signals were applied to the imput of the signal processing systems. Such simulated signals, as shown in figure 6(a), consisted of bursts of different frequencies but equal amplitude. In such a case, the amplitude discriminator would not be able to distinguish between the contributions from each of the two separate phases but the employment of the automatic filter banks in a so-called "slave-operation" permits these two contributions to be separated, as shown in figures 6(b) and 6(c). It is only through the combined employment of different controls such as amplitude discriminator, "slave-control" of the filter banks, photodiode control, etc. that the cross talks between signals from small and large particles can be significantly reduced.

Using the afore-mentioned LDA-equipment, the authors carried out studies of the local velocity distributions of particulate two-phase flows. The measurements were performed in the

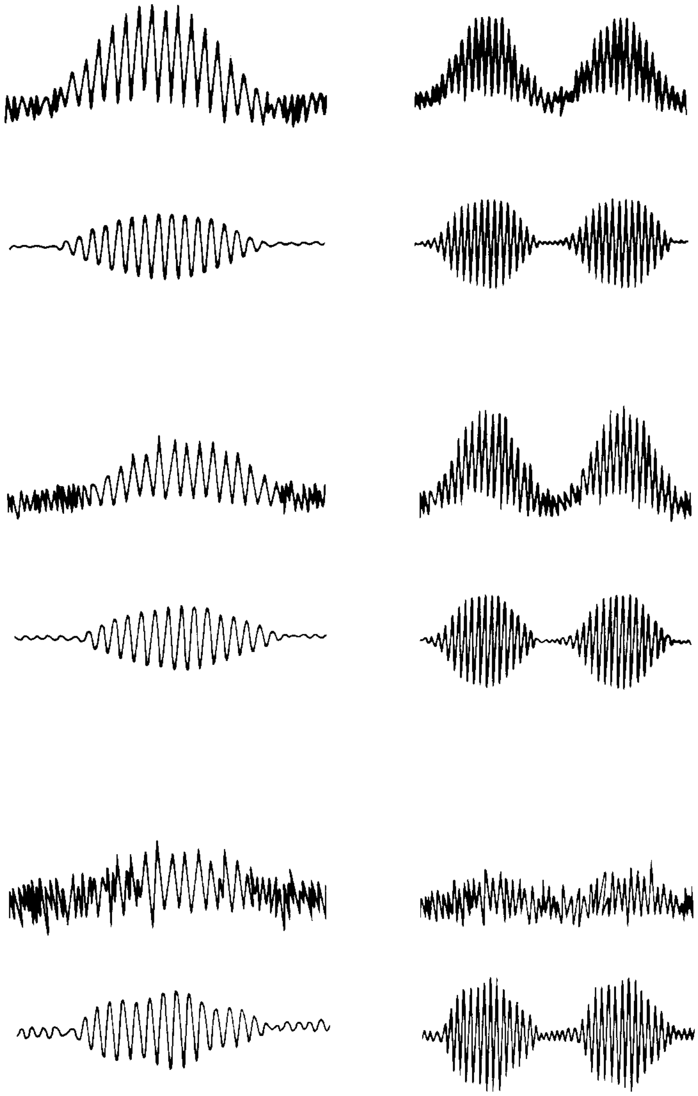


Figure 3. Noisy signals at input and output of automatic filterband.

test section shown in figure 7. The test section consisted of a vertical glass pipe flow of internal radius of 20.9 mm through which filtered air from a regulated pressure system was driven. The supplied air already contained enough minute contaminant particles for the measurement of the air flow in the vertical glass pipe so that no seeding problems existed. Large glass particles of one uniform size were added to this air flow via a Venturi orifice that supplied the needed low pressure to induce the particles into the flow. A wire mesh and a flow straightener ensured an equal distribution of the particles and also helped to regulate the disturbed flow from the Venturi orifice. A cyclon at the end of the test section permitted the glass particles to be separated from and reinjected into the air flow, as indicated in figure 7. Particles of diameters $d_1 = 100\,\mu$, $d_2 = 200\,\mu$, $d_3 = 400\,m$ and $d_4 = 800\,\mu$ were employed in the present experiments.

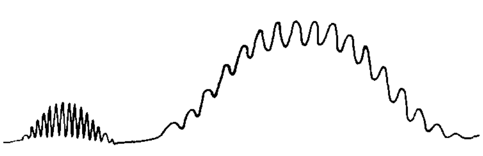


Figure 4. Typical signals from particulate two-phase flow system.

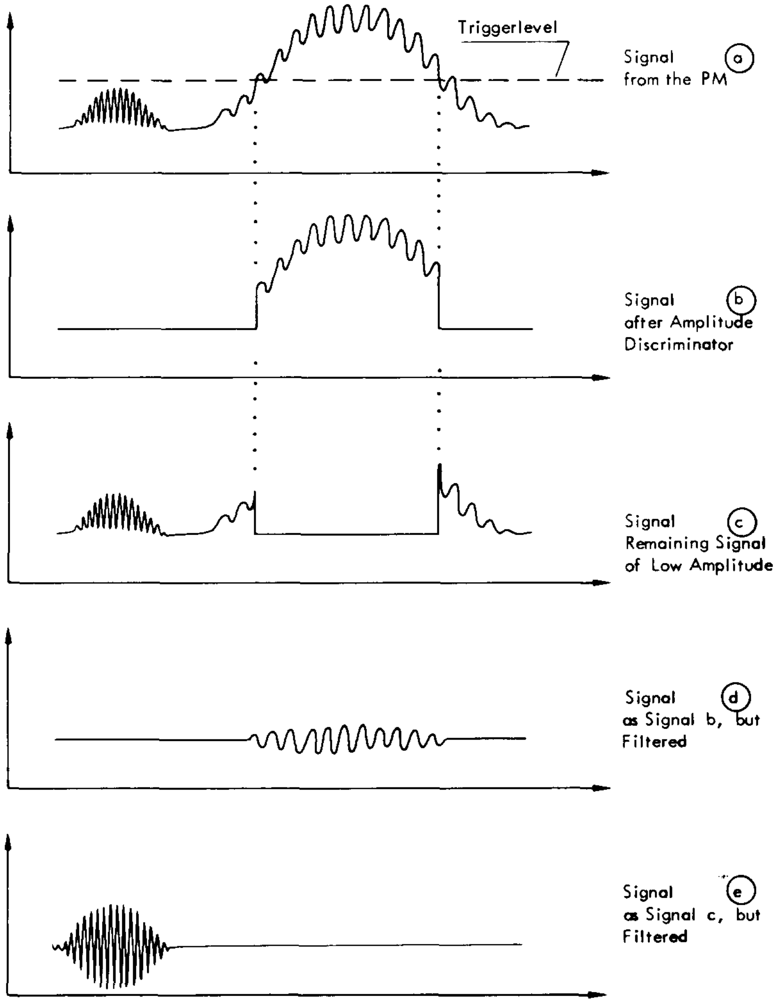


Figure 5. Separation of signals by amplitude discriminator and automatic filterbanks.

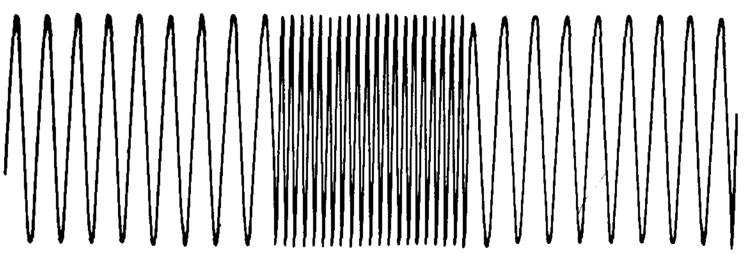


Figure 6(a). Simulated two-phase flow signals; bursts of different frequency.

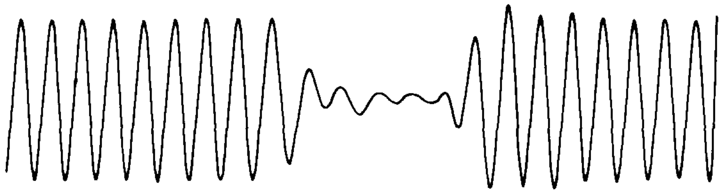


Figure 6(b). Output of automatic filterbank A.

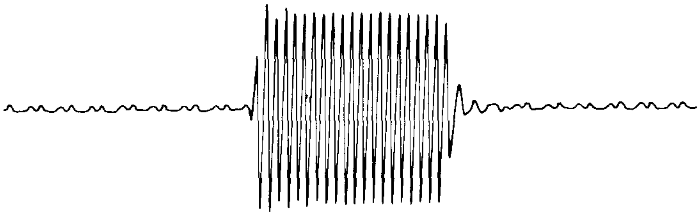


Figure 6(c). Output of automatic filterbank B.

2.2 *Signal processing and data validation*

The processing of laser–Doppler signals by available digital computers became possible with the development of high-speed analog-to-digital converters with large enough storage capabilities and with digitizing rates of 100 MHZ and faster. Devices of this kind permit Doppler bursts to be digitized and stored with sufficient accuracy to allow for meaningful Doppler frequencies to be calculated by digital computers following the A/D-converter store. The evaluation of the Doppler frequency from the recorded signals is controlled by soft-ware programs and, hence, differences in signal qualities can be easily accounted for by introducing suitable data validation schemes.

The A/D-converter which was used in the present study was the 9-bit "Biomation Transient Recorder" Model 8100. This model is particularly suited to record signal bursts as obtained in laser–Doppler anemometry because of its ability to operate in a "delayed-trigger" mode. This

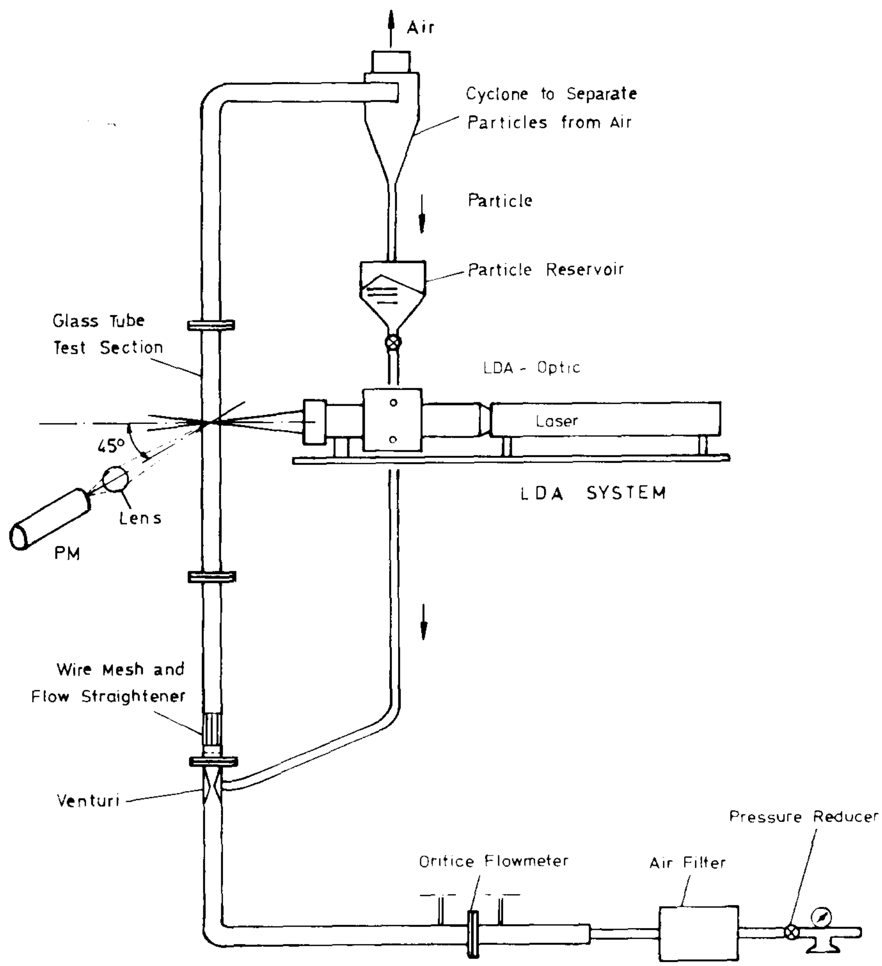


Figure 7. Schematic diagram of the test-section and the LDA-system for two-phase flow measurement.

mode of operation allows the recording of the signals which occur prior to the triggering event. Hence, the recorder can be triggered on the high amplitude peaks in the center of the Doppler bursts and still records the entire bursts in the 2048-word memory of the transient recorder. The Biomation 8100 also offers input attenuation and amplification, and proper adjustment of these input parameters of the transient recorder permits optimal usage of the 8 bits of amplitude resolution.

In the present investigations, the contents of the 2048-word memory were directly transferred to the core memory of an available Hewlett–Packard computer (HP 6116C). Once in the core memory, the data can be either directly analyzed (on-line operation) or stored on disk for further processing to obtain information on the Doppler frequency. Reading the data in and storing them on disk can be carried out at approximately 10,000 words/sec (5 complete memories), and the on-line processing rate depends entirely on the evaluation method, as discussed by Durst & Tropea (1978). If not the entire 2048-word memory is required for analyses then a smaller portion of it may be read thus allowing more Doppler bursts to be scanned per second. However, this modification would mean fewer points for the description of each of the Doppler bursts and its application is, therefore, restricted to the processing of good quality signals. Such signals are usually obtained in model particulate two-phase flows.

The program used in the present study was the "high-speed program" of Durst & Tropea (1978) which has been developed for Doppler signals of high signal-to-noise ratio. This program assumes that there are no multiple zero-crossings in the signal and that every change of the sign of the input signal indicates the presence of another halfcycle of the high-pass filtered Doppler signal. The basic approaches chosen in the high-speed computer program were therefore to calculate the position of the first and last zero-crossings exactly and to count the number of zero-crossings between these end-positions (see figures 8 and 9). Knowing the number of signal cycles and also the time for these zero-crossings to occur, permits the Doppler frequency to be calculated. Bursts that yielded less than 4 cycles between the accurately determined zero-crossings were disregarded.

In order to carry out LDA-measurements with the afore-mentioned program it is necessary for the operator to put in the following information: sample time interval of the transient recorder, the input mode of the program (i.e. on-line or from disk operation), the number of the bursts to be analyzed, the length of the bursts to be used in the program and the two discriminator levels used in the data validation scheme. The program returns the number of accepted bursts, the average frequency and the rms-value of the frequency deviations. The individual frequency values can be stored on a disk for further analysis to obtain probability density distributions and/or to carry out other statistical calculations.

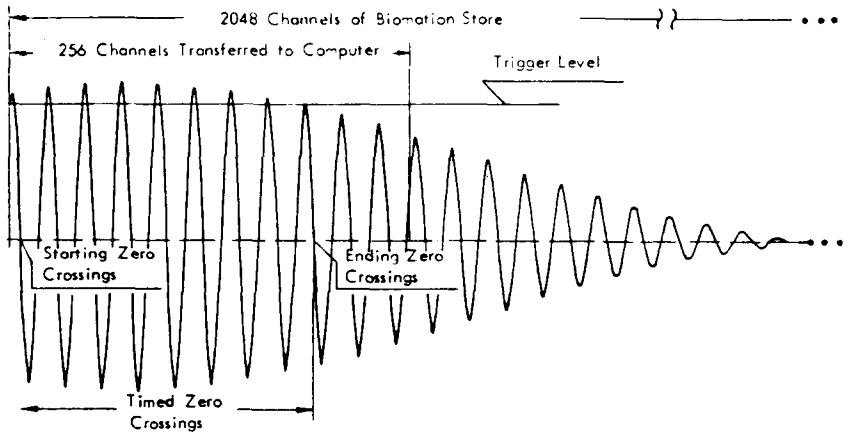


Figure 8. Data transfer and signal validation using the computer program HISPE.

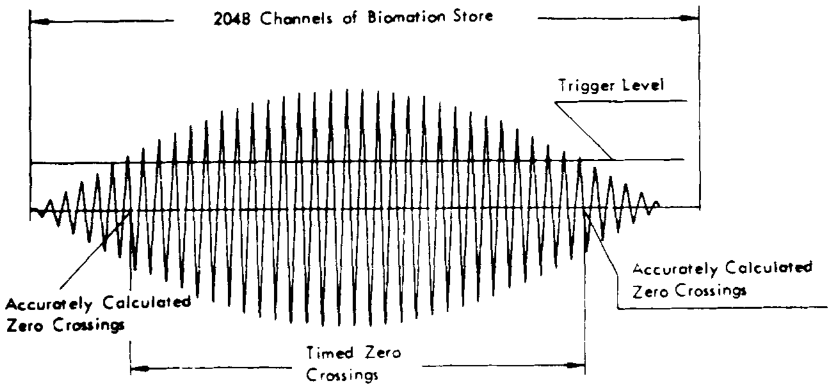
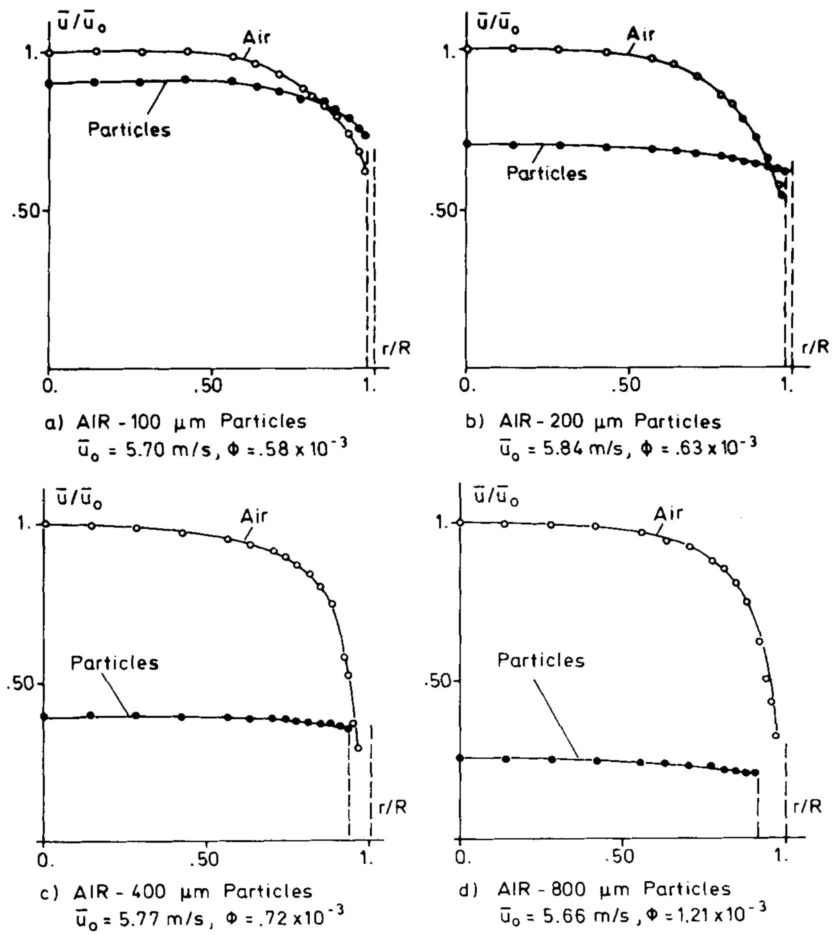


Figure 9. Data transfer and signal validation using the computer program LOSPE.

2.3 *Experimental results*

Figure 10 shows examples of the results obtained in the afore-mentioned test section. Mean velocity distributions are given for the air flow and for the four particle sizes, i.e. for particles



Legends:

$\bar{u}$ - Time-mean axial velocity

$\bar{u}_o$ - Time-mean axial velocity of air on the axis

r - Radial coordinate

R - Pipe inner radius

Ø - Ratio of volume flux of particles to that of air

Figure 10. LDA-measurements in two-phase flows; mean velocity profiles of air and glass spheres in upward direction pipe flow.

of $d_p = 100\ \mu$, $200\ \mu$, $400\ \mu$, and $800\ \mu$. Of the four particle sizes used, for the most part, the particles were found to lag progressively behind the fluid according to their size as expected. However, for the $100\ \mu$ and $200\ \mu$ particles, higher mean velocities were recorded in the near wall region indicating that in this region the particles were leading the fluid. Hence, for the two smaller particles, there existed a radial matching location at which the relative velocity between the fluid and the particles changes its direction from pointing upward in the central region of the pipe to pointing downward in the near wall region. The thickness of this reversed slip-velocity region was about 20% of the pipe radius for the $100\ \mu$ particles and became about 10% of the pipe radius for the $200\ \mu$ particles.

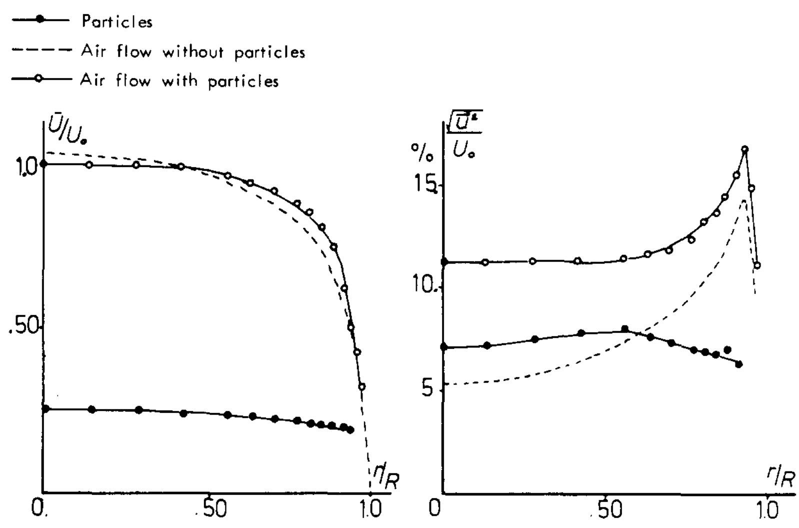
The results also show that the mean velocity profile of the particles becomes close to being constant across the pipe test section as the particle diameter increases. No location was found for the $400\ \mu$ and $800\ \mu$ particles at which the mean fluid velocity and the mean particle velocity took on the same values. Furthermore, with increasing particle diameter a clearly identifiable particle-free region was found near the wall which increased as the particle diameter is increased. This region is apparently much larger than the viscous sublayer thickness of the air flow.

Figure 11 shows the effect which solid particles have on the properties of the air flow. The velocity profiles and turbulence intensity profiles for the axial velocity fluctuations are shown in this figure with and without particles. With particles, the mean velocity profile flattens in the center of the pipe but becomes steeper in the region near the wall. The figure also shows that the turbulence intensities in the air flow are increased due to the presence of the particles. These are typical results for large particles and they indicate the influence of the particle flow on the properties of the fluid flow. For small particles, damping of the turbulence properties has been observed.

3. EXISTING TREATMENTS OF DYNAMICS OF SUSPENSION FLOWS

3.1 *Survey of existing theories*

The lack of combined detailed theoretical and experimental studies and the complexity of the mechanisms of particle and flow interactions have so far prevented the development of a theoretical model which explains the observed deposition rates in physically acceptable terms. Most theoretical treatments adopt the view of a conventional three-layer flow in the vicinity of



$\bar{u}/\bar{u}_o$ = normalized time-mean axial velocity

$u'/\bar{u}_o$ = normalized root mean square axial turbulent fluctuating velocity

Figure 11. LDA-measurements in two-phase upward flow in a vertical pipe. Effects on the air flow due to the presence of 800 u particles.

the wall, the viscous sublayer, the buffer zone and the turbulent core region. In the fully developed turbulent core region particles are assumed to be laterally transported by turbulent diffusion in quite the same way by which scalar quantities such as heat or concentration of species are assumed to be transported in a turbulent stream. Particles reaching the edge of the viscous sublayer as a result of this transport are assumed to coast towards the wall across the sublayer to form deposition.

Lin *et al.* (1953) proposed that for submicron size particles, both the molecular and turbulent diffusivities should be retained in the diffusion equation which govern the transport of particles in boundary-layer flow. However, when the turbulent diffusivity for particles, ϵ_p, was suppressed to zero in the viscous sublayer, the computed deposition on the wall was found to be far below values reported in the available experimental results.

Friedlander & Johnstone (1951) proposed in their study that particles are carried laterally by turbulent diffusivity for the fluid, ϵ_f. The particle transport takes place to within the Stokes stopping distance† of the wall from where the particles travel to the wall by their inertia. In order to bring the computed stopping distance to the same order of magnitude as the sublayer thickness, an unrealistically high value would have to be assumed for the initial transverse velocity of the particles. The value to be chosen for the root mean square value of the transverse fluctuating velocity in the turbulent core must be as high as about 90% of the shear velocity $U_\tau = (\tau_w/\rho_f)^{1/2}$. On the other hand, if the local transverse fluctuating velocity of the fluid $\bar{v}'$ at the edge of the viscous sublayer is used instead, computed despositions based on this scheme will be four orders smaller than the values from measurements for small particles. For larger particles, even with unrealistically large transverse particle velocities assumed, the comparison is worse.

Davis (1966) also introduced an empirical formula for the turbulent diffusivity for particles. He suggested that in computing the Stokes stopping distance, with the proposed modification for ϵ_p, the particle transverse velocity should be assumed to be the root mean square value of the fluid fluctuating velocity $\bar{v}'$ at one stopping distance from the wall rather than that related to the turbulent core. An estimate from this approach, however, would give a deposition more than two orders of magnitude below the experimental data reported by Friedlander & Johnstone (1951).

Beal (1968) introduced in his study the assumption that at the edge of the turbulent core the particle has the same probability of being thrown outward towards the wall as inward back into the turbulent core region. He had to introduce some very unrealistically high transverse velocities for the particles in the calculation of the required stopping distance but managed to produce some improvement in the comparison between theoretical predictions and the experimental results reported by Friedlander & Johnstone (1951). The somewhat arbitrariness of some of the assumptions suggest that Beal's approach will not yield generally acceptable results for different flow situations.

Hutchinson *et al.* (1971) proposed a statistical description of the movement of particles in the turbulent core in which both the fluid and the particles are assumed to have the same and uniform time-mean longitudinal velocity across the whole pipe cross section. With this assumption an analysis is carried out of the random walk of particles due to postulated successive, simple, dynamic interactions of particles with the individual, local, most energetic eddies in the flow. The eddies are taken to have the same average size across the turbulent core region.

The analysis by Hutchinson *et al.* (1971) results in a diffusion equation for the distribution of the mean particle concentration, thus lending philosophical support to the hitherto purely phenomenological concept of a turbulent diffusion controlled particle transport in the core

†The Stokes stopping distance is defined as the distance from the wall to a point in the sublayer where a particle with a specified transverse velocity will be able to reach the wall after overcoming the resistance in the form of the Stokes drag.

region. In this way, the paper demonstrates the usefulness of the concept of particle-flow interactions by just taking into account the most energetic eddies. However, the drawback of the approach by Hutchinson *et al.* (1971) results from the over-simplified flow configuration of uniform time-mean longitudinal velocities for both the fluid and the particles across the core region of the flow. This assumption is not valid for upward flows in which the gravitationally caused otherwise downward motion of the particles causes an inherent velocity difference to exist between the fluid and particle velocity fields. This difference exists also at the outer edge of the core. In addition the thickness of the viscous sublayer as well as a homogeneous boundary condition for the particle concentration would have to be arbitrarily assumed at the edge of the core region. Consequently, results on deposition computed from the particles coasting against the Stokes drag in the viscous sublayer compare favorably with experimental data only with the inclusion of some empirically adjusted constants.

Rouhiainen & Stachiewics (1970) made a significant contribution to an understanding of some of the basic mechanisms of interaction between the particles and the surrounding fluid. They correctly pointed out that the particles' response to the turbulent fluid motions is important for their deposition on the wall. Their examination of the relationship between the particle and the fluid turbulent diffusion coefficients, ϵ_p and ϵ_f respectively, leads to the realization that the usual assumption of ϵ_p being equal to ϵ_f is reasonable only for very small particles. This assumption is found to become questionable for particles of a size of about 1 μ and is completely untenable for particles of a size of 30 μ or larger in the turbulent core and for density ratios of the order of $\rho_p/\rho_f \doteq 1000$.

The theoretical treatment by Rouhiainen & Stachiewics (1970) is based on the concept of the frequency response of a particle in an oscillating flow field. It was first developed by Hjelmfelt & Mockros (1966) and an important consequence of this approach is that the validity of the turbulent diffusion assumption for the particles' transport in a turbulent fluid stream can be characterized by the value of an amplitude ratio, η, which is defined as the ratio of the amplitude of oscillation of the particle to that of the surrounding eddy motion. On the dynamic behavior of a particle in the viscous sublayer, Rouhiainen & Stachiewicz (1970) made the important observation that the classical concept of the Stokes stopping distance cannot be valid, especially in the case of a dense particle passing through the sublayer, since the effect of the shear flow-induced transverse lift force, first derived by Saffman (1965) is no longer negligible. Actually, the importance of the inclusion of this shear flow-induced transverse lift force had already been recognized in the analyses of simple laminar boundary layer flows of a two-phase dispersion by Otterman & Lee (1965, 1970). Their theoretical treatment was subsequently verified experimentally by the use of laser–Doppler anemometry by Lee and Einav (1972) and since then considerable amount of work in this light for more complicated boundary-layer type flows have appeared in the literature, for example Lee *et al.* (1972). Direct experimental verification of this transverse lift force was reported by Rubin (1977).

The main weakness of the analysis by Rouhiainen & Stachiewicz (1970) lies in its lack of a suggestion of a rational scheme to link the two distinctly different flow regions, the turbulent diffusion core and the laminar or quasilaminar viscous layer. This is particularly serious in the determination of the location of demarcation of these two regions for a few obvious reasons. First, since the effect of the shear flow-induced lift force varies sensitively with the location of the particle in the flow field, its trajectory in the viscous layer will depend strongly on the extent of this layer. Secondly, the deposition of the particle also depends strongly on its initial transverse velocity on entering the viscous layer from the turbulent core. It is the fluctuation of the fluid at this demarcation location between the two flow regions which will help to propel the particle sufficiently close to the wall to result in the eventual deposition. Therefore, before realistic predictions of deposition of particles from turbulent streams can be made, the question of the determination of the location of demarcation between the turbulent core and laminar or quasi-laminar viscous layer must first be answered. As already pointed out in the introduction, it is the purpose of this paper to answer this question.

3.2 *Discrepancy between computed deposition from existing theories and directly measured wall deposition*

A common feature of the existing theories of the dynamics of a two-phase suspension flow which leads to wall deposition of particles is their possession of an adjustable empirical factor which is necessary for the achievement of a reasonable comparison between the theoretically predicted and experimentally determined amounts of deposition for a particular flow system. Unfortunately this empirical factor is in no case a universal "correction" constant for all flow systems. For instance, the factor which is determined for a particular particle size and particle to fluid density ratio could make the comparison between theory and experiment on wall deposition as poor as up to four orders of magnitude apart from some other particle size and particle to fluid density ratio. Figure 1 shows one of such comparisons, reported by Wildi (1981). This obvious inconsistency leads to the questioning of the correctness of the physics which has been assumed in these theoretical treatments. Particularly the assumption of the particle's transport by turbulent diffusion, irrespective of its size and the ratio of its density to that of the fluid, in the existing theoretical approaches is to be questioned.

3.3 *Apparent qualitative agreement on mean particle motion between present measurement of turbulent flow and on existing laminar theory for two-phase suspensions*

In the theoretical studies of laminar boundary-layer flows of a two-phase suspension of uniform size particles, since the particles at the edge of the boundary layer have negligible transverse velocity and particles are generally lagging behind the fluid in longitudinal velocity within the boundary layer, the shear flow-induced transverse lift force causes the creation of a low particle number density region adjacent to the wall. This prediction was virtually verified by the presence of a particle free region adjacent to the boundary wall of such flows measured by a two-dimensional loser–Doppler anemometer for the case of a suspension in water of glass particles of a uniform size of from 10 μ to 150 μ, see Lee & Einav (1972).

By including the same shear flow-induced transverse lift force, in addition to the Stokes' drag force, in the study of a particle's behavior in the viscous sublayer at the edge of which the particle has a non-negligible initial transverse velocity, Rouhiainen & Stachiewicz (1970) were able to provide a realistic dynamic mechanism for the deposition of particles on the boundary wall from a turbulent particle suspension flow. In particular, they pointed out the importance of the directional reversal of this lift force on the two sides of the transverse matching position of the longitudinal velocities of the fluid and the particle. For a given main flow in which the particle is lagging behind the fluid in the longitudinal direction, the particle coming into the sublayer initially will experience a combined resistance of drag and lift forces. If the initial transverse velocity of the particle is not high enough for it to reach the matching location of longitudinal velocities of the fluid and the particle, the particle will be kept away from the wall in a way similar to the findings of two-phase laminar boundary layer studies e.g. see Lee & Einav (1972), Lee & Chan (1972). However, if the initial transverse velocity of the particle is high enough for it to pass through this matching location, the lift force will thereafter reverse its direction and propel the particle toward the wall.

In the present experimental studies of turbulent flow of a glass particle–air dispersion having particles of a uniform size in a vertical pipe, for the most part the particles were found to lag progressively behind the fluid according to their size as expected, due to gravity effect, with the exception of the situation existing in the near-wall region for 100 μ and 200 μ particles in which the particles were leading the fluid, as shown in figure 10. In other words, for these smaller size particles, 100 μ and 200 μ, there exists a radial matching location across which the relative velocity of the fluid with respect to the particles changes its direction from pointing upward in the main flow region to pointing downward in the near-wall region. For the two larger size particles, 400 μ, and 800 μ, on the other hand, a clearly identifiable particle-free region was found near the wall and there was no radial location at which the velocities of the fluid and the particles were found to be matched. At first glance, it would seem that all these behaviors could be qualitatively explained by the role played by the shear flow-induced lift force on the particle's trajectory in the viscous sublayer in the theory proposed by Rouhiainen & Stachiewics (1970). However, there is a serious drawback in trying to make this connection. Rouhiainen & Stachiewicz's theory is based on the shear flow within the extremely thin viscous sublayer adjacent to the wall of a thickness of around 0.018 R, where R is the pipe inner radius, for the present case while the thicknesses of the apparent shear region for these particles sizes has been actually many orders of magnitude larger since the radial matching locations for the 100 μ and 200 μ particles are already approximately 0.8 R and 0.9 R respectively and the particle free regions are already of the order of magnitude of 0.10 $R \sim 0.15\,R$ for the 400 μ and 800 μ particles.

On the other hand, apparently, Rouhiainen & Stachiewicz's theory of particle excursion in a viscous shear flow is predicting qualitatively correctly the measured mean particle behavior of the present experimental studies of turbulent flow of a two-phase suspension. The explanation may lie in the scenario that for the present case although the flow around a particle is turbulent, it could actually be considered laminar-like if the particle's dynamic response is already insensitive to the turbulent fluctuations in the flow of the surrounding fluid. Qualitatively the larger and heavier the particle is, the less likely it will still be sensitive to these fluctuations. These observations suggest strongly the usefulness of a new analytical approach to the problem of deposition from a turbulent two-phase flow which includes as basic ingredients the turbulence structure in the flow of the continuous fluid, the size of the particle and ratio of the intrinsic density of the particle to that of the fluid.

4. PROPOSED NEW THEORETICAL APPROACH

4.1 *Particle response to turbulent flow fluctuations*

The present theoretical treatment starts with the Lagrangian governing equation of motion of a spherical particle in a moving turbulent fluid, see Hinze (1959), with the external body force ignored, which was first derived by Tchem (1947):

$$\frac{\mathrm{d}v_p}{\mathrm{d}t} + av_p = av_f + b\,\frac{\mathrm{d}v_f}{\mathrm{d}t} + c\int_0^t \frac{\left[\frac{\mathrm{d}v_f}{\mathrm{d}t'} - \frac{\mathrm{d}v_p}{\mathrm{d}t'}\right]}{(t-t')^{1/2}}\,\mathrm{d}t' \qquad [1]$$

subject to the following restrictions, see Corrsin & Lumley (1957):

$$\frac{\mathrm{d}_p^2}{\nu_f}\frac{\partial v_f}{\partial r} \ll 1 \qquad\qquad \frac{v_f}{\mathrm{d}_p^2\left[\frac{\partial^2 v_f}{\partial r^2}\right]} \gg 1 \qquad [2]$$

where

$$a = \frac{36\mu_f}{(2\rho_p + \rho_f)\mathrm{d}_p^2}$$
$$b = \frac{3\rho_f}{2\rho_p + \rho_f} \qquad [3]$$
$$c = \frac{18}{(2\rho_p + \rho_f)d_p}\left[\frac{\rho_f\mu_f}{\pi}\right]^{1/2}$$

and v_p = velocity of particle in r-direction; v_f = velocity of fluid in r-direction; t = time; d_p = particle diameter; $\nu_f = (\mu_f/\rho_f)$ = kinematic viscosity of fluid; μ_f = dynamic viscosity of fluid; ρ_f = intrinsic density of fluid; ρ_p = intrinsic density of particle.

Expressing v_f and v_p by their Fourier integrals,

$$v_f = \int_0^\infty (\xi \cos \omega t + \lambda \sin \omega t)\,\mathrm{d}\omega$$

$$v_p = \int_0^\infty \{\eta[\xi \cos(\omega t + \beta) + \lambda \sin(\omega t + \beta)]\}\mathrm{d}\omega. \qquad [4]$$

in [1], we obtain the amplitude ratio

$$\eta = [(1 - f_1)^2 + f_2^2]^{1/2} \qquad [5]$$

where

$$f_1 = \frac{\omega[\omega + c(\pi\omega/2)^{1/2}](b-1)}{[a + c(\pi\omega/2)^{1/2}]^2 + [\omega + c(\pi\omega/2)^{1/2}]^2}$$

$$f_2 = \frac{\omega[a + c(\pi\omega/2)^{1/2}](b-1)}{[a + c(\pi\omega/2)^{1/2}]^2 + [\omega + c(\pi\omega/2)^{1/2}]^2} \qquad [6]$$

and ω = frequency of fluid oscillation; ξ, λ = components of amplitude of fluid oscillation; β = phase angle between particle and fluid oscillations.

The amplitude ratio η has been shown to be directly connected to the diffusional characteristics of the particles, e.g. see Hinze (1959), through the following expression:

$$\frac{\epsilon_p}{\epsilon_f} = \frac{\int_0^\infty \eta^2 E_f(n)\,\mathrm{d}n}{\int_0^\infty E_f(n)\,\mathrm{d}n} \qquad [7]$$

where ϵ_p and ϵ_f are the turbulent eddy diffusivities of the particles and the fluid respectively and $E_f(n)$ is the Lagrangian energy spectrum as a function of the wave number n. Thus η gives a measure of the effectiveness of the initiation of particle eddy diffusion caused by the fluid eddy diffusion which is present in the flow. In particular, when $\eta = 1$ and thus $\epsilon_p = \epsilon_f$, the particle's motion is completely controlled by the diffusional motion of turbulent eddies in the surrounding fluid. On the other hand, when $\eta = 0$ and thus $\epsilon_p = 0$, the particles' motion is completely independent of the diffusional motion of turbulent eddies in the surrounding fluid and therefore is totally governed by the quasi-laminar (or laminar, in the case of absence of turbulent eddies) viscous interaction of the mean motion of the surrounding fluid flow field.

From [5], for given values of fluid and particle properties, typically the amplitude ratio η can be plotted against the frequency ω of the eddy motion of the surrounding fluid as shown schematically in figure 12. The characteristics of this plot bring to light the distinctively different behavior of the dynamic response of a particle to the agitation of the ambient fluid in each of the three frequency ranges of eddy motion:

(i) *Turbulent diffusion controlled range*: $\eta = 1$. The particle submits completely to the motion of the eddy and consequently the ensemble of particles in the mixture pursues a diffusional motion dictated by the eddy diffusivity of the surrounding fluid.

(ii) *Mean fluid motion controlled range*: $\eta = 0$. The particle is independent of the diffusional interaction of the eddy motion of the surrounding fluid. Its behavior is dictated by the quasilaminar viscous interaction of the mean motion of the surrounding fluid. The influence of the motion of the eddies in the surrounding fluid on the motion of the particle rests in its

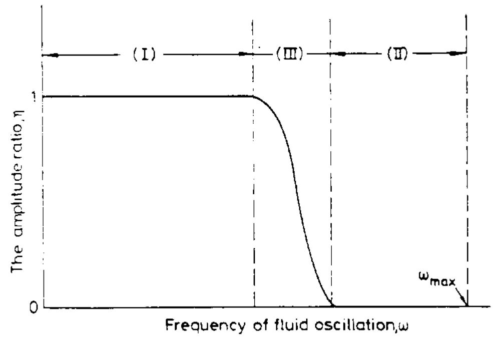


Figure 12. Theoretical result of frequency response of particles.

contribution to the apparent viscosity of the surrounding fluid which governs the dynamic interaction between the particle and the mean motion of the fluid.

(iii) *The intermediate range*: $0 < \eta < 1$. In this range, the motion of the particle is governed neither completely by the diffusional motion of the surrounding fluid nor completely by the quasi-laminar viscous interaction with the mean motion of the surrounding fluid.

4.2 *Particle transport in turbulent wall boundary-layer flows*

The difficulties of applying the preceding results from the analysis of frequency response of a particle to the study of the behavior of particles of a dispersion in turbulent flow next to a wall are two-fold: one is the problem associated with inadequacy of phenomenological analytical tools available for the task in light of the requirements posted by the separate frequency ranges from the analysis and the other the problem associated with the complexity caused by the fact that, in actual flows, both the particles and the eddies usually have a distribution of sizes at any given point and these distributions vary from point to point.

Any theoretical treatment of the particle motion in turbulent dispersion flows has to state how the aforementioned difficulties are circumvented or overcome. The present approach only considers particles of a single size that are exposed to the turbulent flow field which acts on these particles in the three ways described for the frequency ranges (i), (ii) and (iii) in the previous section. Of these three frequency ranges, the turbulent diffusion controlled range (i) and the mean fluid motion controlled range (ii) can be handled by presently available "phenomenological analytical tools". The difficulty rests in finding analytical means to take care of the intermediate range (iii). In an effort to circumvent this difficulty while still preserving the essential features of a phenomenological description of the particles' dynamic behavior according to a frequency scale, the present theoretical treatment proposes a particle frequency response model in which the intermediate range (iii) is suppressed and replaced by extensions from the two remaining ranges as shown in the sketch of figure 13. This suggestion is similar to treatments of electronic filter characteristics in communication theory where the complex filter response is often replaced by a top hat response. In this model, the turbulent diffusion controlled range (i), in which $\eta = 1$, and the mean fluid motion controlled range (ii), in which $\eta = 0$, are brought to join each other at ω_c, the out-off frequency, which is obtained for a particular particle–fluid mixture system by setting an intermediate value for η, say $\eta = \frac{1}{2}$, in [5]. The application of this scheme to the investigation of particle behavior in the two remaining eddy frequency ranges is demonstrated in the following for the sample case of an axissymmetrical turbulent flow of a dispersion of particles of a single size d_p in a vertical circular pipe in which a single value of dominate eddy frequency ω_e can be identified to characterize the turbulent fluid motion at any given radial position. This eddy distribution is taken from existing

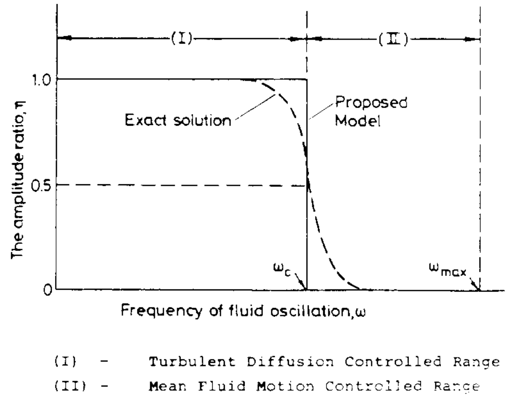


Figure 13. Proposed model of frequency response of particles.

information in turbulent pipe flows and it is shown that, in general, the eddy frequency ω_e increases while the eddy size l_e decreases towards the wall.

For high particle concentrations, the treatment of particle motions in turbulent dispersion flows is forced to adopt a point of view similar to that adopted in the studies of laminar two-phase dispersed flows in which two mutually related and coupled sets of governing equations have been used, one for the fluid motion and the other for the motion of the dispersed particles. To avoid such complications, the present theoretical treatment assumes relatively dilute concentrations of particles and this permits the feed-back from the dynamics of the particles to the motion of the fluid to be neglected, see Dussan & Lee (1969). Hence, in the present treatment, the fluid flow characteristics are assumed to be known and undisturbed by the motion of the particles.

4.3 *Determination of the location of boundary between turbulent diffusion controlled and mean fluid motion controlled regions in a pipe flow*

From the point of view of a particle's frequency response, according to the proposed model, the boundary between the diffusion controlled and the mean fluid motion controlled regions in a turbulent two-phase suspension flow is characterized by a cut-off frequency, ω_c, of the turbulence fluctuations in the surrounding fluid which corresponds to an assigned value of the amplitude ratio of $\eta_c = 1/2$. This conclusion has been reached on the assumption that a single value of dominate eddy frequency, ω_e, can be identified to characterize the turbulent motion at any given point in the flow in spite of the fact that the turbulent fluid motion is actually characterized by a distribution of eddy frequencies. This distribution in frequency can be related to a distribution in eddy size through, in this case, the local transverse velocity.

From a consideration of the turbulent kinetic energy of a distribution of eddy sizes, Townsend (1957) developed an expression for the most energetic eddy size, l_e, of a turbulent flow. Using Laufer's (1953) experimental data for fully developed turbulent pipe flows, Hutchinson *et al.* (1971) evaluated this most energetic eddy size, l_e, as a function of radial location for two values of flow Reynolds number that are an order of magnitude apart. These data are given in a non-dimensional form of l_e/R vs r/R in figure 14 where R is the pipe inner radius and r the radial coordinate. In general, this figure shows that the value of l_e/R stays fairly constant in much of the central portion of the pipe and starts to drop off towards the wall from a radial position of $r/R = 0.6$; the plot for the lower Reynolds number flow shows that the value of l_e/R drops off much faster. In order to relate this most energetic eddy to its counterpart in frequency, we can make use of Laufer's (1953) data on the root mean square of the transverse turbulent fluctuating velocity, $\bar{v}'$. Figure 14 also provides these experimental data in a non-dimensional form as $(\bar{v}'/U_\tau)$ where $U_\tau = (\tau_w/\rho_f)^{1/2}$ is the friction velocity for the fluid and τ_w the wall shear stress. Using this information, one can obtain the most energetic fluctuating

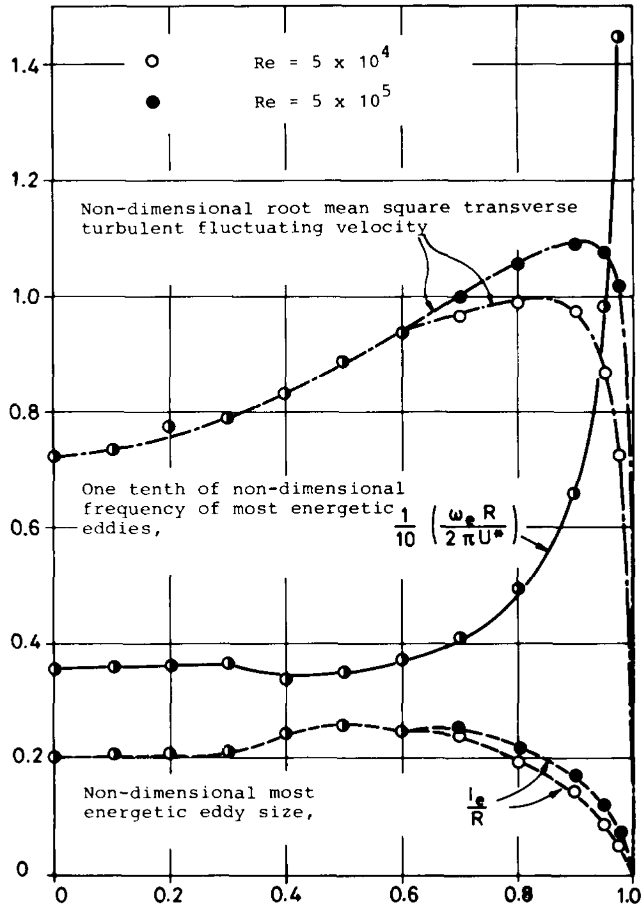


Figure 14. Frequency of most energetic eddies in a fully developed turbulent pipe flow.

frequency, ω_e, by dividing the roof mean square transverse velocity, $\bar{v}'$, by the most energetic eddy size, l_e, for every radial location in the flow. Results of such calculations are presented in the non-dimensional plot of $(\omega_e R/U_\tau)$ vs r/R of figure 14. The most significant feature of this plot is that, in the range of $r/R = 0.6$ to $r/R = 1$, the non-dimensional most energetic fluctuating frequency $(\omega_e R/U_\tau)$, is a function only of the radial location while both l_e/R and $(\bar{v}'/U_\tau)$ are found to be sensitive functions of flow Reynolds number.

Based on the proposed model of the particle's dynamic response, the joining boundary between the turbulent eddy diffusion controlled frequency range and the mean fluid motion controlled frequency range is specified by a selected intermediate value of the amplitude ratio of $\eta = \frac{1}{2}$. By setting $\eta = \frac{1}{2}$ in [5] and calling the frequency therein the cut-off frequency, ω_c, we have after some manipulation,

$$(N_s)_c^4 + \frac{1}{(2)^{1/2}}(N_s)_c^3 + 3(N_s)_c^2 + \frac{(16S+11)}{324(2)^{1/2}}(N_s)_c - \frac{(4S^2+4S-35)}{3,888} = 0 \qquad [8]$$

where N_s is the Stokes number,

$$N_s = \left[\frac{\nu_f}{\omega d_p^2}\right]^{1/2}$$

and

$$(N_s)_c = \left[\frac{\nu_f}{\omega_c d_p^2}\right]^{1/2},$$

the Stokes number corresponding to the cut-off frequency ω_c.

For a given value of the density ratio S, where $S = \rho_p/\rho_f$, the positive real root of [8] yields the value of $(N_s)_c$, the Stokes number corresponding to the cut-off frequency ω_c. For instance, in a water droplet–air system, $S = 1000$, the solution to [8] yields $(N_s)_c = 5.10$.

The cut-off frequency ω_c of a particle's dynamic response is related to a Lagrangian description of the fluctuating motion of the fluid surrounding the particle while the frequency of the most energetic eddy in a flow, ω_e, is related to an Eulerian description of the fluid motion as measured by a stationary velocity sensing instrument. Strictly speaking, these two frequencies, ω_c and ω_e, cannot be related to each other without the knowledge of the velocity of the particle. In the case of the transverse motion of particles in a turbulent flow of a two-phase suspension in a vertical pipe, however, this conceptual difficulty may be rendered comparatively less serious. The mean transverse particle velocity in this type of flow is, for most situations, much lower than the transverse fluctuating velocity of the fluid and the mean transverse fluid velocity is virtually zero. When a particle's transverse motion is dominated by the mean motion rather than fluctuating motion of the surrounding fluid, the Lagrangian and the Eulerian descriptions of the fluctuating motion of the surrounding fluid therefore can be expected to be close to each other. On the other hand, when a particle submits completely to the fluctuating motion of the surrounding fluid, the Lagrangian description of the fluctuating motion of the surrounding fluid differs from its Eulerian counterpart mainly in amplitude rather than frequency. If the primary interest here is to study a particle's dynamic response in terms of the amplitude ratio as a function of the frequency of agitation of the surrounding fluid, the final result should be relatively insensitive to the choice of either of these descriptions for the input to the analysis.

Based on the preceding discussions, the location of the joining boundary between the turbulent eddy diffusion controlled core region and the annular mean fluid motion controlled region is then determined from the following matching condition:

$$\omega_e = \omega_c. \tag{9}$$

Hence, the most energetic fluctuating frequency, ω_e, for the flow is obtained from figure 14 as a function of the radial location for a given pipe flow and the cut-off frequency, ω_c, from the solution of [8] for a given particle size d_p and physical properties of the particle and fluid. Using condition [9] a radial location is determined and defined as the cut-off radius r_c. Results of this matching are shown in the plot of the non-dimensional cut-off radius r_c/R against the non-dimensional particle diameter $\tilde{d}_p = d_p \cdot (N_s)_c \cdot (2\pi U_\tau/\nu_f R)^{1/2}$ in figure 15. Therefore, for a given flow system there is a fixed cut-off radius r_c for a fixed particle size d_p. In general r_c/R decreases from a value of unity with the increase of d_p until the nondimensional particle diameter reaches a value of about 0.528 beyond which $r_c = 0$.

The sketches in figure 16 compare the different classifications of flow regimes in a fully developed turbulent pipe flow. The top sketch shows the conventional single-phase fluid flow regime classification of three fixed transverse regions: the turbulent core, the buffer zone and the viscous sublayer. The middle sketch shows the conventional particle transport flow regime classification of two fixed transverse regions: the turbulent diffusion core and the "viscous sublayer". The outer edge of this layer is loosely put at somewhere between the buffer zone and the viscous sublayer in the conventional single-phase fluid flow classification.

The bottom sketch shows the present frequency response based particle transport flow regime classification of two transverse regions: the turbulent diffusion controlled core region and the mean fluid motion controlled quasi-laminar region. The boundary between these two regions is characterized by the cut-off radius, r_c, which is, according to the above analysis, a function of the particle size, d_p, and the flow and physical system properties.

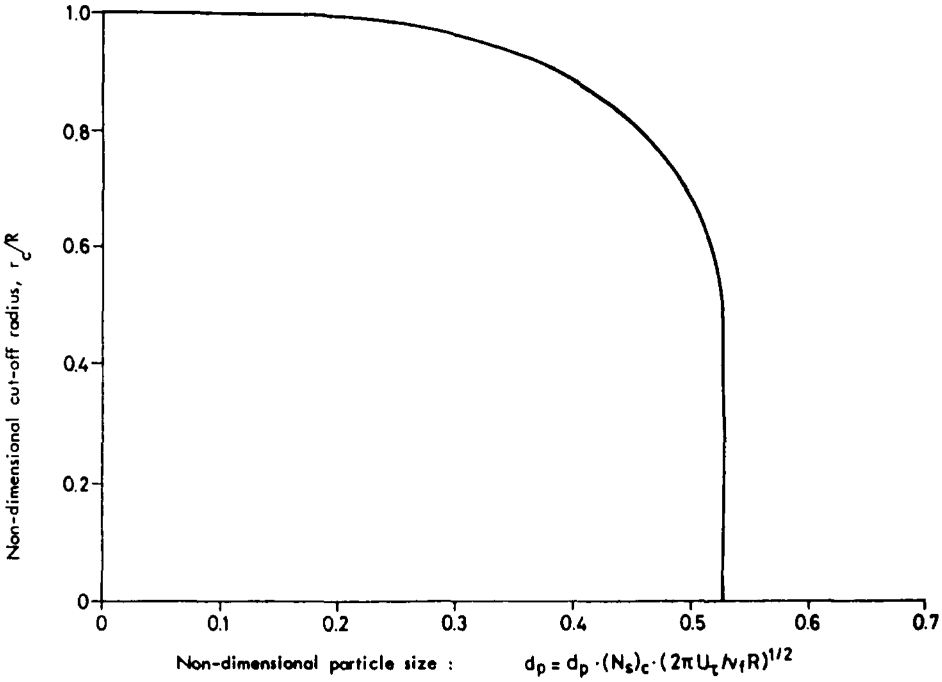


Figure 15. The cut-off radius for particles in a fully developed turbulent pipe flow.

4.4 A qualitative explanation of the measured main feature of turbulent flow fo a two-phase suspension in a vertical pipe by the present theoretical approach

According to the present analysis which is based on the frequency response of a particle, for each particle size there is a particular value of the cut-off radius r_c which separates the turbulent diffusion controlled core and the mean fluid motion controlled quasi-laminar annular region as shown in figure 15. In the case of the flow under discussion, this cut-off radius becomes zero at a particle size of only 20 μ, and will remain zero for larger sizes. In other words, for the range of particle sizes involved in the experiments described in section 2, 100 μ, 200 μ, 400 μ and 800 μ, there is no turbulent diffusion controlled core and as far as these

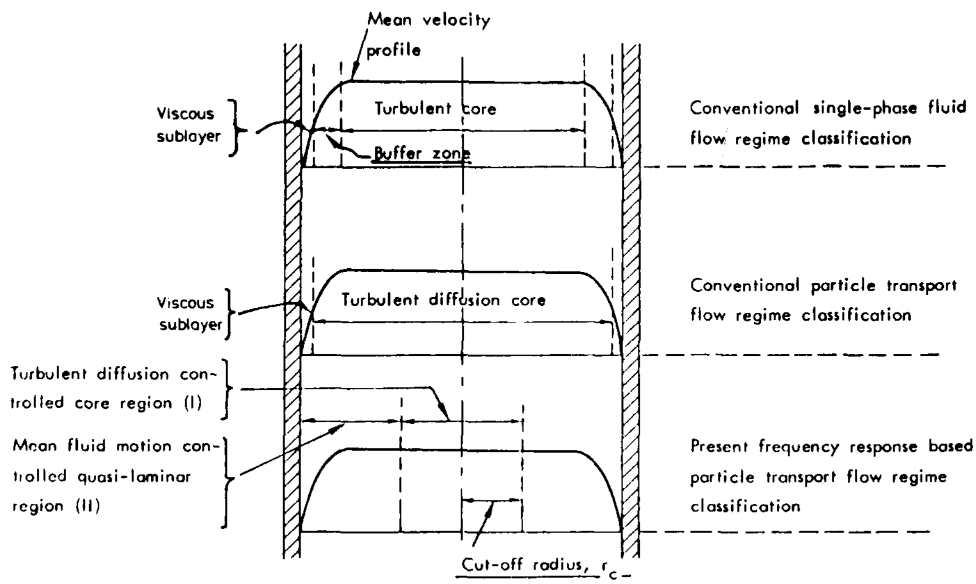


Figure 16. Particle transport flow regime classification in fully developed turbulent pipe flow.

particles are concerned, the whole flow across the total pipe cross section can be considered quasi-laminar and controlled by the mean fluid motion. With this rational justification, the role played by the generalized shear flow-induced lift force of the present analysis then can be properly used to provide a qualitative explanation to the above-mentioned behaviors of particles in a pipe flow.

5. CONCLUSIONS AND FINAL REMARKS

The present paper reports an experimental investigation of the turbulent flow of a two-phase suspension with one uniform-size particles in a vertical pipe by the use of laser Doppler anemometry (LDA) technique. The results of measurements on the mean velocities of the two phases bring to light certain seemingly peculiar behaviors of the particles which defy the prediction of the conventional analyses of turbulent two-phase suspension flows. Nevertheless, these particle behaviors can be qualitatively explained by an existing theory of particle excursion in a laminar shear flow. One of the reasons for this may be that for the present case, although the flow around a particle is turbulent, it could actually be considered laminar-like if the particle's dynamic response is already insensitive to the turbulent fluctuations in the flow of the surrounding fluid. These observations strongly suggest a new theoretical approach to the problem of the turbulent flow of a two-phase suspension which is based on the concept of the dynamic response characteristics of the particles to the motion of the surrounding eddies.

In the accompanying theoretical analysis, an approximation to the resultant frequency response of a particle yields a cut-off frequency beyond which the particle no longer responds to the turbulent flow fluctuations. By matching this cut-off frequency with the frequency of the motion of the most energetic eddies which is deduced from existing experimental results of the corresponding single-phase flows, a demarcation line has been found between regions in which the particle transport is controlled entirely by turbulent diffusion and by properties of the mean flow field respectively. According to this theory, for the range of particle size covered, there is already no turbulent diffusion controlled region left for the transport of particles in the present experimental investigation. In other words, as far as the particles are concerned, the whole flow field can be considered laminar-like. This theoretical finding thus provides a qualitative explanation to the apparently laminar-like behaviors of the turbulent flow of a two-phase suspension which has been discovered in the present experimental investigation.

In addition, although the present analysis is developed for particles of one single size, it can be readily extended to a dispersion of particles of multiple sizes as long as the basic assumption concerning the diluteness of the particles is still valid. For each particle size d_p, there is a particular cut-off frequency ω_c which then, by assuming $\omega_c = \omega_e$, can be used to obtain the corresponding cut-off radius r_c for that particle size as shown in figure 17. The behavior of particles of this size can be determined from the framework of the present analysis according to the cut-off radius for that particle size. The behavior of particles of multiple sizes are then

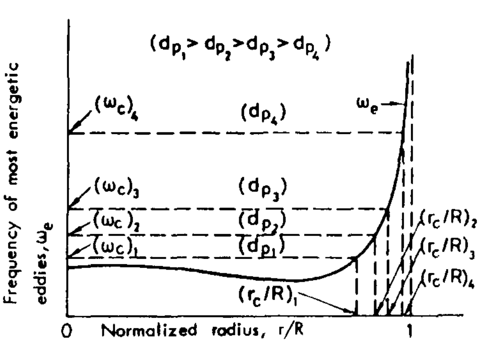


Figure 17. Determination of cut-off radius for multiple size particles in fully developed turbulent pipe flow.

simply the superposition of results obtained from individual sizes present in the dispersion. In general, the turbulent diffusion controlled core region is bigger and consequently the mean fluid motion controlled quasi-laminar annular region is smaller for smaller particles. This point is also qualitatively verified by the results of a recent experiment on the local measurement of turbulent upward flow of a dilute water droplet–air two-phase dispersion in a vertical rectangular channel, see Srinivasan & Lee (1978).

Acknowledgements—This work was carried out at the Sonderforschungsbereich 80 of the Universität Karlsruhe and at the Department of Mechanical Engineering of the State University of New York at Stony Brook. The authors gratefully acknowledge the generous supports of the U.S. Nuclear Regulatory Commission, the Deutsche Forschungsgemeinschaft and the Alexander von Humboldt Foundation,

The authors would also like to express their thanks to Drs P. Hutchinson, J. Srinivasan and A. K. Rastogi and Dipl.-Math. B. Schönung for their helpful discussions on the various respects of this work, to Messrs. J. Heidbreder and M. Zaré for their assistance in the experiment, and to Mrs. M. Licata & Mrs. H. Sy for their valuable contribution to the preparation of the manuscript.

REFERENCES

Alexander, L. G. & Coldren, C. L. 1951 Droplet transfer from suspending air to duct wall. *Ind. Engng Chem.* **45**, 1325–1331.

Beal, S. K. 1968 Transport of particles in turbulent flow to channel or pipe walls. Bettis Atomic Power Laboratory Report No, WAPDTM-765, Westinghouse Electric Corp., Pittsburg, U.S.A.

Cousins, L. B. & Hewitt, G. G. 1968 Liquid phase mass transfer in annular two-phase flow: droplet deposition and liquid entrainment. U.K.A.E.A. Report No. R-5657, Harwell, Great Britain,

Corrsin, S. & Lumley, J. L. 1957 On the equation of the motion of a particle in turbulent fluid. *Appl. Scientific Res.* Section A, **6**, 114–116.

Davis, C. N. 1966 Deposition of aerosols through pipes, *Proc. Roy. Soc.* (London), Series A **289**, 235–246.

Di Giovanni, P. R. & Lee, S. L. 1974 Impulsive motion in a particle fluid suspension including particulate volume density and migration effects. *J. Appl. Mech.* Series E, **41**, 35–41.

Durst, F. 1978 Studies of particle motion by Laser–Doppler techniques. *Proceedings of Dynamic Flow Conference*, Marseille, France.

Durst, F. & Heidbreder, J. 1978 Eine automatische Filterbank für LDA-Signale. Report No. SFB 80/M/140, Universität Karlsruhe, Fed. Rep. Germany.

Durst, F. & Tropea, C. 1978 Processing of laser–Doppler signals by means of a transient recorder and a digital computer. Report No. SFB 80/E/118, Universität Karlsruhe.

Durst, F. & Zaré, M. 1975 Laser Doppler measurement in two-phase flows. *Proceedings of the LDA Symposium*, pp.403–429. Copenhagen, Denmark.

Dussan, E. B. & Lee, S. L. 1969 Behaviour of spherical solid particles released in a laminar boundary layer along a flat plate. *Appl. Scientific Res.* **20**, 465–477.

Friedlander, S. K. & Johnstone, H. F. 1957 Deposition of suspended particles from turbulent gas streams. *Ind. Engng Chem.* **49**, 1151–1156.

Ganic, E. N. & Rohsenow, W. M. 1976 On the mechanism of liquid drop deposition in two-phase flow. *The American Society of Mechanical Engineers Winter Annual Meeting*, Paoer No. 76-WA/HT-18. New York, U.S.A.

Hinze, J. O. 1959 *Turbulence*, 2nd Ed. McGraw-Hill, New York, U.S.A.

Hjelmfelt, A. T., Jr. & Mokros, L. F. 1966 Motion of discrete particles in a turbulent fluid. *Appl. Scientific Res.* **16**, 149–161.

HUTCHINSON, P., HEWITT, G. F. & DUKLER, A. E. 1971 Deposition of liquid or solid despersion from turbulent gas streams: a stochastic model. U.K.A.E.A., Report No. AERE R 6637, Harwell, 1970; also *Chem. Engng Sci.* **26**, 419–439.

KNEEN, T. & STRAUSS, W. 1969 Deposition of dust from turbulent gas streams. *Atmospheric Environment* **3**, 55–67.

LAUFER, J. 1953 The structure of turbulence in fully developed pipe flow. NACA Technical Note No. 2954.

LEE, S. L. & CHAN, W. K. 1972 Two-phase laminar boundary layer along a vertical flat wall. *Proceedings of the Hydrotransport* 2 *Conference*, Paper A4. University of Warwick, Coventry, Great Britain.

LEE, S. L. & EINAV, S. 1972 Migration in a laminar boundary layer measured by the use of a two-dimensional laser-Doppler anemometer. *Prog. Heat Mass Transfer* **59**, 385–403.

LIN, C. S., MOULTON, R. W. & PUTNAM, G. L. 1953 Mass transfer between solid wall and fluid streams. *Ind. Engng Chem.* **45**, 667–678.

LIU, B. & AGARWAL 1974 Experimental observation of aerosol deposition in turbulent flow. *J. Aerosol Sci.* **5**, 145–155.

OTTERMAN, B. & LEE, S. L. 1969 Particle migrations in laminar mixing of a suspension with a clean fluid. *Zeitschrift für Angewandte Mathematik und Physik (ZAMP)* **20**, 730–749.

OTTERMAN, B. & LEE, S. L. 1970 Particulate velocity and concentration profiles for laminar flow of a suspension over a flat plate. *Proceedings of the* 1970 *Heat-Transfer and Fluid Mechanics Institute*, pp. 311–322. Stanford Press.

ROUHIAINEN, P. O. & STACHIEWICZ, J. W. 1970 On the deposition of small particles from turbulent streams. *J. Heat Transfer* **92**, 169–177.

RUBIN, G. 1977 Widerstands- und Auftriebsbeiwerte von ruhenden, kugelförmigen Partikeln in stationären, wandnahen laminaren Grenzschichten. Doctoral Dissertation, Universität Karlsruhe, Fed. Rep. Germany.

SAFFMAN, P. Q. 1965 The lift on a small sphere in a slow shear flow. *J. Fluid Mech.* **22**, 385–400.

SRINIVASAN, J. & LEE, S. L. 1978 *Measurement of Turbulent Dilute Two-Phase Dispersed Flow in a Vertical Rectangular Channel by Laser Doppler Anemometry, Measurement of Polyphase Flows*, (Edited by D. E. STOKE). American Society of Mechanical Engineers.

TCHEN, C. M. 1947 Mean value and correlation problems connected with the motion of small particles suspended in a turbulent field. Ph.S. Thesis, Delft, Netherlands.

TOWNSEND, A. A. 1957 *The Structure of Turbulent Shear Flow*. Cambridge University Press, Cambridge, Great Britain.

WELLS, A. C. & CHAMBERLAIN, A. C. 1967 Transport of small particles to vertical surfaces. *Br. J. Appl. Phys.* **18**, 1793–1799.

WILDI, J. 1981 Doctoral Dissertation, ETH-Zurich, in preparation.